

Available online at [www.sciencedirect.com](http://www.sciencedirect.com)**ScienceDirect**

Procedia Engineering 154 (2016) 1439 – 1447

**Procedia  
Engineering**[www.elsevier.com/locate/procedia](http://www.elsevier.com/locate/procedia)

12th International Conference on Hydroinformatics, HIC 2016

## Improving the efficiency of Cellular Automata for sewer network design optimization problems using Adaptive Refinement

M. H. Afshar<sup>a</sup>, M.M. Zaheri<sup>a</sup>, J.H. Kim<sup>b</sup><sup>a</sup> School of Civil Engineering & Enviro-Hydroinformatic COE, Iran University of Science and Technology, , Tehran, Iran.<sup>b</sup> School of Civil, Environmental and Architectural Engineering, Korea University, Seoul, South Korea.

---

### Abstract

This paper introduces an adaptive procedure to improve the efficiency of a two phase simulation-optimization cellular automata algorithm recently proposed by the authors for the optimal design of household sewer networks. In the proposed method, the continuous decision variables are discretized to turn the original mixed-integer problem to a discrete problem which is then solved by a two-phase CA method. It is obvious that coarse discretization requires low computational effort but may lead to sub-optimal solution while fine discretization may produce better solutions at the expense of higher computational cost. An adaptive refinement approach is, therefore, proposed to reduce the computational cost of the CA method with no adverse effect on the quality of the final solution. The optimization process starts with coarse discrete values of pipes nodal elevations and the problem is solved for optimal solution. A finer discretization of the pipe nodal elevations is then constructed in the neighborhood of optimal pipes nodal elevations obtained from the first run and the same process is used to find the new solution. This process is continued until no change in the solution is possible. The proposed method is applied to solve two benchmark problems of literature. The result explicitly shows that the proposed adaptive refinement approach leads to quality solution with much reduced computational effort.

© 2016 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Peer-review under responsibility of the organizing committee of HIC 2016

**Keywords:** Sewer Network; Adaptive Refinement; Cellular Automata; SWMM; Optimization

---

### 1. Introduction

Sewer networks are one of the most important infrastructures in urban areas, the absence of which may cause some important health and environmental problems for citizens. Construction of these networks is costly and, therefore, trying to build networks with lower construction cost is an essential issue for designers. Excavation and pipe installation costs are two main parts of sewer networks construction costs. These costs are a function of pipe slope

and pipe diameter with conflicting nature. The aim of any optimization process, therefore, can be thought as crating an optimal balance between these parameters. Due to the complexity of this problem, different optimization methods have been used for optimal design of sewer networks such as Non-Linear Programming (NLP), Linear Programming (LP), Dynamic Programming (DP), Discrete Differential Dynamic Programming (DDDP), heuristic methods and more recently Cellular Automata (CA) algorithm.

Merritt and Bogan [1] proposed a DDDP and solved the sewer optimization problem with the pipe diameter and nodal elevations as decision variables. Dajani and Hasit [2] used a hybrid LP to design the sewer network in which LP produced continues range of pipes diameter and commercial diameter was selected by a mixed integer LP. Mays and Yen [3] used DP and DDDP as a serial method for optimal design of a tree-shape drainage system. Robinson and Labadie [4] presented CSUDP-SEWER based on DP and designed three different drainage systems. Gupta, et al. [5] developed a DP method for optimal design of gravity sewerage system. A new approach of DP was proposed by Kulkarni and Khanna [6] for optimal design of sewer networks. Miles and Heaney [7] proposed a heuristic method on spreadsheet templates for optimal design of sewer network and compared the results with other existing method based on DP. Li and Matthew [8] presented a hybrid method for layout and size optimization of sewer networks in which a searching direction method and DDDP were used to find the optimal layout and pipe sizes, respectively. Liang, et al. [9] applied Genetic Algorithm (GA) to find the optimal configuration of pipes diameter in sewer systems with the lowest possible cost construction. Afshar [10] developed an adaptive Ant Colony Optimization Algorithm (ACOA) for optimal design of sewer networks. Afshar, et al. [11] used a GA as optimizer and EPA's Storm Water Management Model (SWMM) as simulator for optimal design of storm water collecting systems. Afshar [12] used a rebirthing Particle Swarm Optimization (PSO) algorithm to design storm sewer networks with lowest construction cost. Afshar [13] proposed two Constrained and Unconstrained versions of Continuous ACOA for optimal design of storm sewer networks. Moeini and Afshar [14] used a hybrid ACOA-TGA (Tree Growing Algorithm) for optimal layout and size determination of sewer networks in which TGA was used to construct feasible layouts while the optimal network was selected by the ACOA.

More recently CA has been used as an efficient optimization algorithm for optimal design of sewer networks. The first application of CA to sewer network design problems is due to Guo, et al. [15] who proposed a series of ad-hoc updating rule to update the cell states. The method, however, suffered from severe limitation of requiring pre-defined pipe slopes. Afshar, et al. [16] presented a CA with mathematically derived updating rule for optimal design of household wastewater networks. The nodal elevation was considered as the cell state and the optimal pipe diameter was calculated using the maximum allowable flow ratio. Afshar and Rohani [17] considered the nodal elevations and pipe diameters as decision variables of problem and proposed a Hybrid Cellular Automata (HCA) for optimal design of sewer networks. Afshar and Rohani [18] proposed a hybrid GA-GHCA method for optimal design of pumped sewer networks in which GA found the position of pump stations while GHCA was used for optimal sizing of the network. Afshar and Rohani [19] proposed GHCA method for optimal design of both gravity and pumped sewer networks. More recently, Zaheri and Afshar [20] coupled a Two Phase CA with EPA's SWMM and presented a two phase simulation-optimization Cellular Automata algorithm for optimal design of household sewer networks. Pipe nodal elevations and pipes diameter were considered as decision variables updated by an ad-hoc updating rules in parallel.

## 2. SEWER NETWORK OPTIMIZATION PROBLEM

The main goal in optimal design of sewer network is to find the pipe sizes and slopes which leads to a sewer system of least cost. Excavation and pipe installations costs are the main part of construction costs which are functions of the pipe slopes and pipe diameters. These design parameters are in conflict with each other and, therefore, the aim of any optimization process is to create an optimal balance between them. The objective function of the sewer network optimization problem can be formulated as:

$$\text{Min } C_T = C_d + \sum_{i=1}^{NL} L_i K_p(D_i X_i) + \sum_{i=1}^{NN} K_m(H_i) \quad (1)$$

Here  $C_T$  is total cost of the network,  $C_d$  is pump and drop costs,  $NL$  is the number of pipes in the network,  $L_i$  is the length of the  $i^{\text{th}}$  pipe,  $K_p$  is the unit cost for the  $i^{\text{th}}$  pipe,  $D_i$  is the diameter of  $i^{\text{th}}$  pipe,  $X_i$  is the average excavation depth of the  $i^{\text{th}}$  pipe which is a function of the pipe nodal elevations,  $K_m$  is the manhole cost function,  $NN$  is the number of nodes in

the network, and  $H_i$  is the depth of  $i^{\text{th}}$  manhole.

Some typical constraints of the sewer network design with operational purposes can be considered as:

$$V_{\min} \leq V_l \leq V_{\max} \quad l: 1, \dots, NL \quad (2)$$

Where  $V_l$  is the velocity of  $l^{\text{th}}$  pipe,  $V_{\min}$  and  $V_{\max}$  are the allowable minimum and maximum velocity, respectively.

$$(\beta)_{\min} \leq (\beta)_l \leq (\beta)_{\max} \quad l: 1, \dots, NL \quad (3)$$

Where  $\beta_l$  is ratio of the flow depth to diameter of  $l^{\text{th}}$  pipe,  $\beta_{\min}$  and  $\beta_{\max}$  are the allowable minimum and maximum ratio of flow depth to pipe diameter, respectively.

$$X_{\min} \leq X_l^i, X_l^j \leq X_{\max} \quad l: 1, \dots, NL \quad (4)$$

Where  $X_l^i, X_l^j$  are the upstream and downstream cover depths of  $l^{\text{th}}$  pipe, respectively, and  $X_{\min}$  and  $X_{\max}$  are the allowable minimum and maximum cover depths. This constraint also can be rewritten as:

$$h_{\min} \leq h_l^i, h_l^j \leq h_{\max} \quad l: 1, \dots, NL \quad (5)$$

Where  $h_l^i, h_l^j$  are the nodal elevation of  $l^{\text{th}}$  pipe, and  $h_{\max}$  and  $h_{\min}$  are the allowable maximum and minimum nodal elevations, respectively.

$$D_l \in \mathbf{D} \quad l: 1, \dots, NL \quad (6)$$

Where  $D_l$  is the diameter of  $l^{\text{th}}$  pipe and  $\mathbf{D}$  is the set of diameter of existing commercial pipes.

$$D_i \geq \bar{D}_i \quad i: 1, \dots, NN \quad (7)$$

Where  $D_i$  is the outlet pipe diameter of  $i^{\text{th}}$  manhole and  $\bar{D}_i$  is the set of inlet pipes diameters of  $i^{\text{th}}$  manhole.

$$q_l \geq Q_l \quad l: 1, \dots, NL \quad (8)$$

Where  $q_l$  is the flow capacity of  $l^{\text{th}}$  pipe and  $Q_l$  is the design discharge of  $l^{\text{th}}$  pipe. Flow capacity of each pipe depends on the roughness and hydraulic radius of it.

### 3. TWO PHASE SIMULATION-OPTIMIZATION CA METHOD FOR OPTIMAL DESIGN OF SEWER NETWORKS

In the two phase simulation-optimization cellular automata, the optimization problem is divided into two sub-problems with different decision variables which are solved iteratively using a two-stage CA method (Zaheri and Afshar [20]). The optimization process starts with arbitrary values of pipe diameters and pipe nodal elevations. A brief description of the method is given below.

#### First Stage:

In the first stage, the pipes diameter are fixed and the pipes nodal elevations are considered as decision variable of the optimization problem. The network nodes are considered as CA cells and nodal elevations of the pipes meeting at the node as the cell states and the upstream and downstream pipes connected to the node are considered as the neighborhood of the cell. A series of ad-hoc local updating rules based on engineering judgment aimed at minimizing the objective function and satisfaction of the operational constraints is derived and used for updating the cell states. The updating rules are applied in parallel on each of the cells. The process of updating is continued until the convergence is achieved. The network hydraulic conditions including flow velocity in the pipes and the ratio of flow depth to pipe diameter at each CA iteration are calculated using the kinematic wave routing model of EPA's SWMM.

#### Second Stage:

In the second stage, the pipes diameter are considered as decision variables of optimization problem while the pipe nodal elevations obtained from the first stage are fixed. The network pipes are, therefore, considered as the CA cells with zero neighborhood. Once again, a series of ad-hoc local updating rules based on engineering judgment aimed at minimizing the pipe installation cost and satisfaction of operational constraints are derived and used to update all pipe diameters in

parallel. The updating process of pipes nodal elevations and pipe diameters are continued iteratively until the convergence is achieved.

#### 4. PROPOSED ADAPTIVE REFINEMENT APPROACH

The two phase simulation-optimization cellular automata algorithm for optimal design of sewer networks requires that the pipe nodal elevations are in a discrete form. This requires that the allowable ranges of pipe nodal elevations are discretized so that the proposed CA method can be used. Discretization size has a decisive effect on the performance of the algorithm, namely, the quality of CA solutions and the computational cost required. Coarse discretization saves the computational cost, but reduces the chance of reaching the optimal solution, while finer discretization increases the computational cost of the method which could be preventive when solving large scale problems.

In this article, an adaptive refinement approach is proposed to reduce the computational cost of the CA method while maintaining its ability to find near optimal solutions. The proposed process starts with a coarse discretization size for the allowable range of pipe nodal elevations and the CA method is used to obtain a solution. In the next run, the allowable range of pipe nodal elevations is restricted to the neighborhood of the obtained solution, discretization size is halved and a second solution obtained using the CA method. The process of reducing the allowable range of pipe nodal elevations and the discretization size is continued until convergence is achieved.

#### 5. MODEL APPLICATION

In this section, the performance of adaptive CA is investigated by designing two benchmark sewer networks from the literature. In both examples, only 10 discrete points was used to discretize the allowable range of the pipe nodal elevations. The first example is a sewer network originally designed by Mays and Wenzel [21]. This sewer system consists of 20 pipes and 21 nodes and its layout is shown in Figure 1. The characteristics of the network are provided in Table 1. Meredith [22] cost function is used to calculate the cost of excavation and pipe installation.

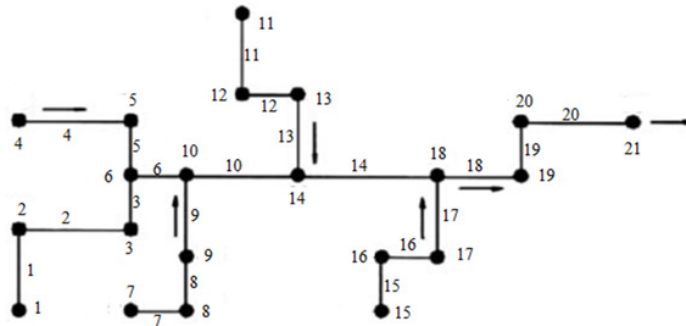


Figure 1. Network layout of the first example

$$K_p = \begin{cases} 10.98D + 0.8X - 5.98 & \text{if } D \leq 3' \text{ and } X \leq 10' \\ 5.94D + 1.166X + 0.504XD - 9.64 & \text{if } D \leq 3' \text{ and } X \geq 10' \\ 30D + 4.9X - 105.9 & \text{if } D > 3' \end{cases} \quad (9)$$

$$K_m = 250 + h_m^2 \quad (10)$$

Where  $K_p$  is the unit cost of pipe installation (\$/ft),  $D$  is the pipe diameter (ft),  $X$  is the average excavation depth (ft),  $K_m$  is the cost of manhole construction (\$) and  $h_m$  is the manholes depth (ft). The network is constrained to a maximum velocity of 12 (fps), minimum velocity of 2 (fps), maximum ( $\frac{V}{d}$ ) of 0.82, minimum ( $\frac{V}{d}$ ) of 0.1 and minimum cover depth of 8 (ft). It's assumed that pipes have a constant Manning roughness coefficient with the value of 0.013.

Mays and Wenzel [21] presented a DDDP for optimal design of this network. Robinson and Labadie [4] proposed a DP

while Miles and Heaney [7] proposed a heuristic method on spreadsheet templates for its solution. Afshar [10] applied an adaptive refinement with ACOA and a re-birthing particle swarm optimization algorithm (RPSO) (Afshar [12]) to solve this problem. Recently some CA-based algorithms are proposed and used for optimal design of this network. Afshar, et al. [16] used a single stage CA method, Afshar and Rohani [17] applied a two-stage hybrid Cellular Automata (HCA) and more recently Afshar and Rohani [19] proposed a General Hybrid Cellular Automata (GHCA) to solve this problem. In a new work, Zaheri and Afshar [20] presented the Two Phase Simulation-Optimization Cellular Automata to design this network.

Table 1. Data of the first example

Pipe	Ground elevations (ft.)		Length (ft.)	Design discharge (cfs)
	Upstream	Downstream		
1	500	495	350	4
2	495	487	400	7
3	487	480	350	9
4	490	485	400	4
5	485	480	430	8
6	480	470	550	22
7	490	485	500	8
8	485	475	450	12
9	475	470	350	16
10	470	465	500	44
11	485	475	500	9
12	475	470	350	16
13	470	465	350	20
14	465	455	565	71
15	468	464	400	4
16	464	460	300	6
17	460	455	345	9
18	455	451	400	87
19	451	448	500	89
20	448	445	612	94

Table 2. Maximum, minimum and average costs obtained over 10 runs using proposed Adaptive CA for the first example

Method		Total cost			SSD	NOFE	
		Max	Min	Average			
Afshar [12]	Conventional PSO	309,000	246,003	265,667	0.0809	30,000	
	Re-birthing PSO	286,444	242,889	256,611	0.0622	30,000	
Afshar, et al. [16]	CA method	269,334	253,484	264,883	0.0238	43	
Afshar and Rohani [17]	Discrete HCA	300,186	247,412	270,912	0.0560	39	
	Continuous HCA	276,826	248,100	257,681	0.0329	6	
Zaheri and Afshar [20]	Two-Phase CA	256,760	240,084	250,814	0.0208	515	
	1	255,964	240,238	248,393	0.0220	174	
Proposed adaptive method	NORS	2	255,324	239,855	248,008	0.0210	184
	3	255,168	239,757	247,910	0.0210	192	

SSD: Scaled Standard Deviation

NOFE: Number of Function Evaluation

NORS: Number of Refinement Steps

The proposed CA method is used to solve this problem and the results are compared with existing solutions. Since the initial guessed solutions could have significant effect on the quality of CA results, 10 runs with different initial randomly generated solutions was carried out to assess the sensitivity of the proposed adaptive method to the initial solution. Table 2 compares the maximum, minimum, average solution cost and corresponding standard deviation obtained by adaptive CA in 10 run with different initial guessed solution to other existing solutions. The results show that sensitivity of the proposed adaptive CA to initial guessed solution is much less than other methods. Table 3 presents the optimal network cost and number of function evaluation required by different methods. The results show that proposed adaptive refinement process reduces the computational cost the method while improving the quality of the results. Details of the optimal solution obtained by the proposed method are presented in Table 4.

The second example is a part of Kerman sewerage system in Iran consisting of 20 pipes and 21 nodes as shown in Figure

2. The characteristics of this network are in Table 5. The cost function of excavation and pipe installation is as follows:

$$K_p = 1.93e^{3.43D} + 0.812X^{1.53} + 0.437DX^{1.47} \quad (11)$$

$$K_m = 41.46h_m \quad (12)$$

Where  $K_p$  is the unit cost of pipe installation (\$/m.),  $D$  is the pipe diameter (m),  $X$  is the average excavation depth (m),  $K_m$  is the cost of manhole construction and  $h_m$  is the manholes depth.

This network is constrained to a maximum velocity of 3 (m/s), minimum velocity of 0.3 (m/s), maximum ( $\frac{V}{d}$ ) of 0.82, minimum ( $\frac{V}{d}$ ) of 0.1 and minimum cover depth of 2.45 (m). It's assumed that the pipes have a constant Manning roughness coefficient with the value of 0.013. Mansouri and Khanjani [23] were the first to design this network using mathematical programming and GA. Afshar, et al. [16] used one stage CA method while Afshar and Rohani [17] applied two-stage HCA method to design this network. In a resent work, Zaheri and Afshar [20] used a Two Phase Simulation-Optimization CA to optimally design this sewer network.

Table 3. Optimal network cost obtained by different methods for the first example

Method		Cost (US\$)	NOFE
Mays and Wenzel [21]		265,775	-
Robinson and Labadie [4]		275,218	-
Miles and Heaney [7]		245,874	-
Afshar, et al. [11]	GA-TRANS2	244,747	-
Afshar [10]	Conventional ACO	245,991	72,522
	ACO with refinement	241,513	27,625
Afshar [12]	Conventional PSO	246,003	30,000
	Re-birthing PSO	242,889	30,000
Afshar, et al. [16]	CA method	253,484	43
Afshar and Rohani [17]	Discrete HCA	247,412	39
	Continuous HCA	248,100	6
Afshar and Rohani [19]	Discrete GHCA	241,845	8
	Continuous GHCA	246,892	10
Zaheri and Afshar [20]	Two-Phase CA	240,084	515
Proposed adaptive method		239,757	192

Table 4. Results obtained by proposed Adaptive CA method for the first example

pipe	Diameter (inch)	Crown elevations (ft.)		Maximum velocity (ft/s)	$\frac{V}{D}$
		Upstream	Downstream		
1	12	492.00	487.00	6.16	0.77
2	15	487.00	479.00	8.20	0.66
3	15	479.00	472.00	8.48	0.81
4	12	482.00	476.95	5.81	0.82
5	18	477.00	472.00	6.95	0.62
6	24	472.00	462.00	10.57	0.63
7	18	482.00	477.00	6.54	0.65
8	18	477.00	467.00	9.77	0.66
9	21	467.00	462.00	8.83	0.71
10	30	462.00	456.24	10.22	0.82
11	15	477.00	467.00	8.48	0.81
12	21	467.00	462.00	8.83	0.71
13	24	462.00	457.00	9.42	0.64
14	36	454.06	447.00	12.00	0.78
15	12	460.00	454.95	5.81	0.82
16	15	455.20	452.00	6.15	0.74
17	18	452.00	447.00	7.77	0.62
18	42	447.00	443.00	11.76	0.72
19	42	443.00	439.08	10.56	0.82
20	42	439.08	433.57	11.31	0.81

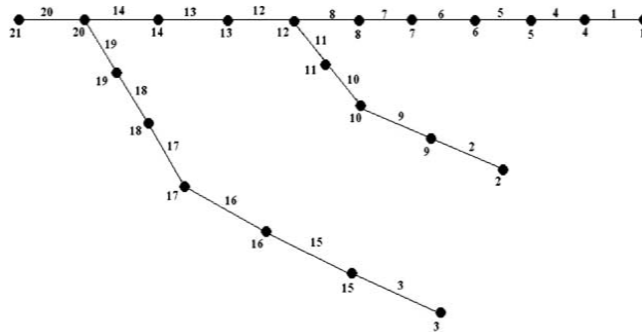


Figure 2. Network layout of the second example

Table 5. Data of the second example

pipe	Ground elevations (m)		Length (m)	Design discharge (lps)
	Upstream	Downstream		
1	74.59	73.66	260	27.9
2	70.70	69.90	300	54.9
3	73.00	71.50	400	21.1
4	73.66	72.10	460	30.4
5	72.10	71.19	260	32.4
6	71.19	69.85	300	34.0
7	69.85	68.24	450	36.6
8	68.24	67.28	400	38.7
9	69.90	69.30	270	56.2
10	69.30	68.40	310	58.0
11	68.40	67.28	440	59.6
12	67.28	66.22	470	96.7
13	66.22	65.82	350	101.2
14	65.82	65.42	340	104.7
15	71.50	70.10	400	26.4
16	70.10	68.60	400	30.0
17	68.60	66.80	500	31.9
18	66.80	66.10	400	40.3
19	66.10	65.42	590	44.6
20	65.42	64.50	320	165.9

Table 6. Maximum, minimum and average costs obtained over 10 runs using adaptive CA methods for the second example

Method		Total cost			SSD	NOFE	
		Max	Min	Average			
Afshar and Rohani [17]	Discrete HCA	82,870	77,327	79,472	0.0238	45	
	Continuous HCA	78,026	77,433	77,967	0.0024	38	
Zaheri and Afshar [20]	Two-Phase CA	82,326	76,750	78,828	0.0260	1184	
Proposed adaptive method	NORS	1	81,772	77,342	79,018	0.0220	134
		2	81,791	77,285	78,983	0.0220	196

Table 6 compares the maximum, minimum, average solution cost and corresponding standard deviation obtained by proposed adaptive CA method in 10 runs with different randomly guessed solution to other existing solutions. Once again, the results show that the sensitivity of the proposed method to initial guessed solution is negligible. Table 7 shows the optimal network cost and number of function evaluation required by different existing methods. Details of the optimal solution obtained by the proposed method are also presented in Table 8. The results show that proposed adaptive refinement process leads to near optimal solution with much less computational cost than alternative methods and the Two-Phase simulation-Optimisation method of Zaheri and Afshar [20].

Table 7. Optimal network cost obtained by different methods for the second example

Method		Cost (US\$)	NOFE
Mansouri and Khanjani [23]		83,116	-
Mansouri and Khanjani [23]	GA	77,736	100,000
Afshar, et al. [16]	CA method	80,879	23
Afshar and Rohani [17]	Discrete HCA	77,327	45
	Continuous HCA	77,433	38
Zaheri and Afshar [20]	Two-Phase CA	76,750	1184
Proposed adaptive method		77,285	196

Table 8. Result obtained by proposed adaptive CA method for the second example

pipe	Diameter (mm)	Crown elevations (m.)		Maximum velocity (m/s)	$\frac{V}{D}$
		Upstream	Downstream		
1	250	72.3900	71.4600	0.80	0.67
2	350	68.6000	67.8000	0.85	0.63
3	200	70.7500	69.0944	0.77	0.82
4	250	71.4600	69.9000	0.80	0.73
5	250	69.9000	68.9900	0.81	0.76
6	250	68.9900	67.6500	0.91	0.71
7	300	67.7000	66.0900	0.87	0.58
8	300	66.0900	65.1300	0.75	0.69
9	300	67.7500	66.8315	0.91	0.82
10	400	66.9315	66.3500	0.76	0.58
11	350	66.3000	65.1800	0.85	0.68
12	400	65.2300	64.1700	0.90	0.81
13	450	64.2200	63.7756	0.73	0.82
14	500	63.8256	63.4700	0.70	0.71
15	250	69.1444	67.9000	0.75	0.67
16	250	67.9000	66.4000	0.83	0.69
17	250	66.4000	64.6000	0.82	0.74
18	350	64.7000	64.0000	0.68	0.59
19	350	64.0000	63.3200	0.58	0.74
20	400	63.3700	61.3370	1.51	0.82

## 6. CONCLUDING REMARKS

An adaptive refinement procedure was proposed in this article to improve the efficiency of the two phase simulation-optimization cellular automata for optimal design of sewer network problems. In the proposed method, the continuous decision variables are discretized to turn the original mixed-integer problem to a discrete problem which is then solved by a two-phase CA method. It is obvious that coarse discretization requires low computational effort but may lead to sub-optimal solution while fine discretization may produce better solutions at the expense of higher computational cost. An adaptive refinement approach is, therefore, proposed to reduce the computational cost of the CA method with no adverse effect on the quality of the final solution. The optimization process starts with a coarse discrete value of pipes nodal elevations. A finer discretization of the pipe nodal elevations is then constructed in the neighborhood of optimal pipes nodal elevations obtained from the first run and the same process is used to find the new solution. This process is continued until no change in the solution is possible. The proposed method is applied to solve two benchmark problems of literature. The result explicitly shows that the proposed adaptive refinement approach leads to quality solution with much reduced computational effort.

## References

- [1] L. B. Merritt and R. H. Bogan, "Computer base optimal design of sewer systems," *Journal of environmental Eng.*, vol. 99, pp. 35-53, 1973.
- [2] J. S. Dajani and Y. Hasit, "Capital cost minimization of drainage networks," *journal of environmental eng.*, vol. 100, 1974.



- [3] L. W. Mays and B. C. Yen, "Optimal cost design of branched sewer systems," *Water resources research*, vol. 11, pp. 37-47, 1975.
- [4] D. K. Robinson and J. W. Labadie, "Optimal design of urban storm water drainage system," *International symposium on urban hydrology, hydraulics, and sediment control, University of Kentucky Lexington*, pp. 145-156, 1981.
- [5] J. M. Gupta, S. L. Mehndiratta, and M. J. Khanjani, "Gravity wastewater collection systems optimization," *Journal of environmental Eng*, vol. 109, pp. 1195-1209, 1983.
- [6] V. S. Kulkarni and P. Khanna, "Pumped wastewater collection systems optimization," *Journal of Environmental Engineering*, vol. 111, pp. 589-601, 1985.
- [7] S. W. Miles and J. P. Heaney, "Better than optimal method for designing drainage systems," *Journal of Water Resources Planning and Management, ASCE*, vol. 114, pp. 477-499, 1988.
- [8] G. Li and R. G. S. Matthew, "New approach for optimization of urban drainage systems," *Journal of Environmental Engineering* vol. 116, pp. 927-944, 1990.
- [9] L. Y. Liang, R. G. Thompson, and D. M. Young, "Optimizing the design of sewer networks using genetic algorithms and tabu search," *Engineering, Construction and Architectural Management* vol. 11, pp. 101-112, 2004.
- [10] M. H. Afshar, "Improving the efficiency of ant algorithms using adaptive refinement: Application to storm water network design," *Advances in Water Resources*, vol. 29, pp. 1371-1382, 2006.
- [11] M. H. Afshar, A. Afshar, M. A. Mariño, and A. A. S. Darbandi, "Hydrograph-based storm sewer design optimization by genetic algorithm," *Canadian Journal of Civil Engineering* vol. 33, pp. 319-325, 2006.
- [12] M. H. Afshar, "Rebirthing particle swarm optimization algorithm: application to Storm Sewer Network design," *Canadian Journal of Civil Engineering*, vol. 35, pp. 1120-1127, 2008.
- [13] M. H. Afshar, "A parameter free continuous ant colony optimization algorithm for the optimal design of storm sewer networks," *Advances in Engineering Software* vol. 41, pp. 188-195, 2010.
- [14] R. Moeini and M. H. Afshar, "Constrained Ant Colony Optimisation Algorithm for the layout and size optimisation of sanitary sewer networks," *Urban Water Journal* pp. 1-20, 2012.
- [15] Y. Guo, G. A. Walters, S. T. Khu, and E. Keedwell, "A novel cellular automata based approach to storm sewer design," *Engineering Optimization* vol. 39, pp. 345-364, 2007.
- [16] M. H. Afshar, M. Shahidi, M. Rohani, and M. Sargolzaei, "Application of cellular automata to sewer network optimization problems," *Scientia Iranica, Sharif University Of Technology*, vol. 18, pp. 304-312, 2011.
- [17] M. H. Afshar and M. Rohani, "Optimal design of sewer networks using cellular automata-based hybrid methods: Discrete and continuous approaches," *Engineering Optimization*, vol. 44, pp. 1-22, 2012.
- [18] M. H. Afshar and M. Rohani, "GA–GHCA model for the optimal design of pumped sewer networks," *Canadian Journal of Civil Engineering*, vol. 42, pp. 1-12, 2014.
- [19] M. H. Afshar and M. Rohani, "A General Hybrid Cellular Automata Algorithm for the Optimal Design of Gravity/Pumped Sewer Networks," *International Journal of Pressure Vessels and Piping under review*, 2015.
- [20] M. M. Zaheri and M. H. Afshar, "A Two Phase Simulation-Optimization Cellular Automata Method for Sewer Network Design optimization," *Applied Soft Computing, under review*, 2016.
- [21] L. W. Mays and H. G. Wenzel, "Optimal design of multi-level branching sewer systems," *Water Resources Research*, vol. 12, pp. 913-917, 1976.
- [22] D. D. Meredith, "Dynamic programming with case study on planning and design of urban water facilities," *Treaties on urban water systems, Colorado State Univ., Fort Collins*, 1972.
- [23] M. R. Mansouri and M. J. Khanjani, "Optimization of sewer networks using nonlinear programming," *Journal of Water and Wastewater (in Persian)*, pp. 20-30, 1999.